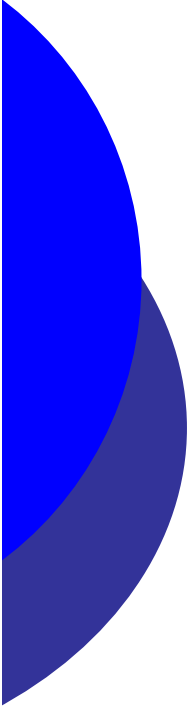
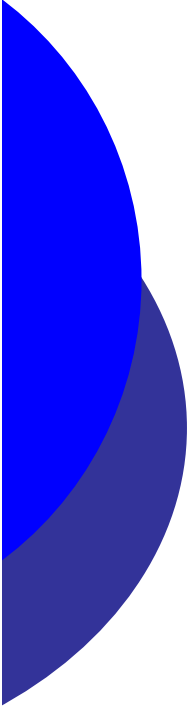


Project management



Project Scheduling: PERT/CPM

- ⊕ **Project Scheduling with Known Activity Times**
- ⊕ **Project Scheduling with Uncertain Activity Times**
- ⊕ **Considering Time-Cost Trade-Offs**



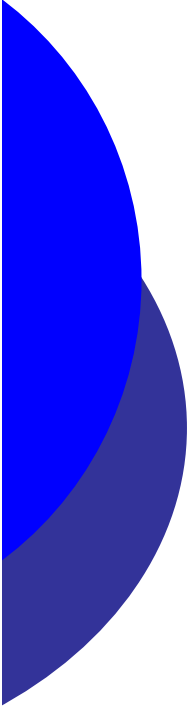
Introduction to Project Management

⊕ **Project Scheduling or project management is used to schedule, manage and control projects which are comprised of various independent activities or tasks.**

⊕ **Example: Building a New Home**

When building a home individual subcontractors are hired to:

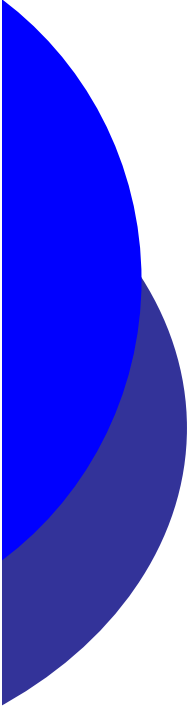
- **Grade and prepare the land**
- **Build the foundation**
- **Frame up the home**
- **Insulate the home**
- **Wire (Electricity, Cable, Telephone lines) the home**
- **Drywall**
- **Paint (inside)**
- **Put vinyl siding on home**
- **Install Carpet**
- **Landscape**
- **Lay Concrete**



Introduction to Project Management

Home builders must answer several questions

- ⊕ What is the total time required to complete the project if no delays occur?
- ⊕ When do the individual activities (subcontractors) need to start and finish?
- ⊕ Which subcontractors will delay the earliest completion date if falls behind its schedule (i. e. the critical activities)?
- ⊕ For other activities, how much delay can be tolerated?
- ⊕ Project Management (PERT/CPM) will help us to address all of the above issues.



PERT/CPM



PERT

- Program Evaluation and Review Technique
- Developed by U.S. Navy for Polaris missile project
- Developed to handle uncertain activity times

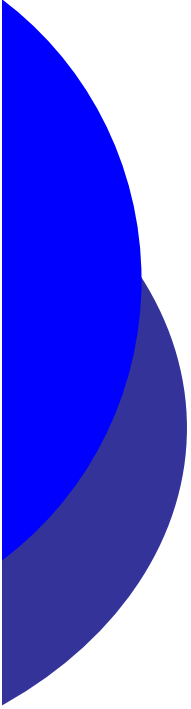


CPM

- Critical Path Method
- Developed by Du Pont & Remington Rand
- Developed for industrial projects for which activity times generally were known



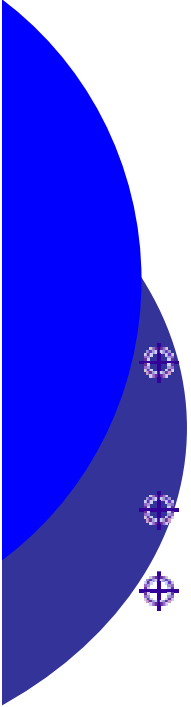
Today's project management software packages have combined the best features of both approaches.

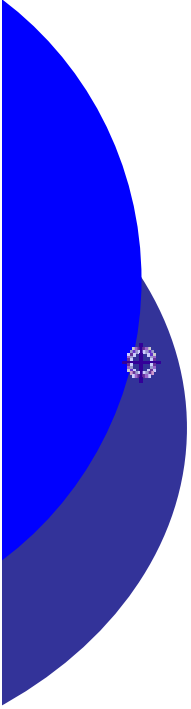


PERT/CPM

- ⊕ **PERT and CPM have been used to plan, schedule, and control a wide variety of projects:**
 - **R&D of new products and processes**
 - **Construction of buildings and highways**
 - **Maintenance of large and complex equipment**
 - **Design and installation of new systems**

PERT/CPM

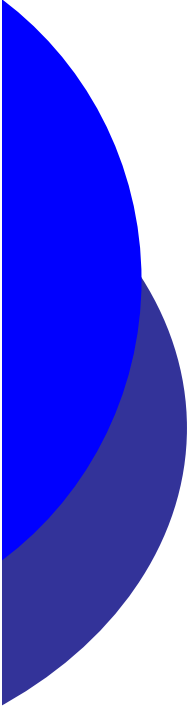
- 
- PERT/CPM is used to plan the scheduling of individual activities that make up a project.
 - Projects may have as many as several thousand activities.
 - ⊕ A complicating factor in carrying out the activities is that some activities depend on the completion of other activities before they can be started.



PERT/CPM

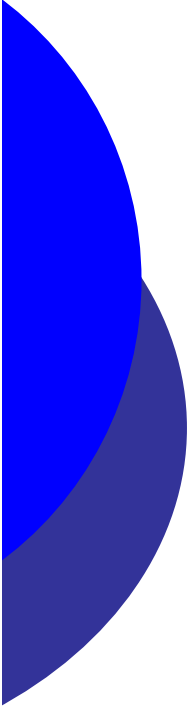
Project managers rely on PERT/CPM to help them answer questions such as:

- What is the total time to complete the project?
- What are the scheduled start and finish dates for each specific activity?
- Which activities are critical and must be completed exactly as scheduled to keep the project on schedule?
- How long can noncritical activities be delayed before they cause an increase in the project completion time?



Project Network

- ⊕ A project network can be constructed to model the precedence of the activities.
- ⊕ The nodes of the network represent the activities.
- ⊕ The arcs of the network reflect the precedence relationships of the activities.
- ⊕ A critical path for the network is a path consisting of activities with zero slack.
- ⊕ Slack is the amount of time that noncritical activities can be delayed without increasing the project completion time.
- ⊕ Immediate predecessor(s) is (are) activities that must be completed immediately before the current activity can begin.



The Critical Path

- ⊕ A path through a network is one of the routes following the arrows (arcs) from the start node to the finish node.
- ⊕ The length of a path is the sum of the (estimated) durations of the activities on the path.
- ⊕ The (estimated) project duration or project completion time equals the length of the longest path through the project network.
- ⊕ This longest path is called the critical path. (If more than one path tie for the longest, they all are critical paths.)

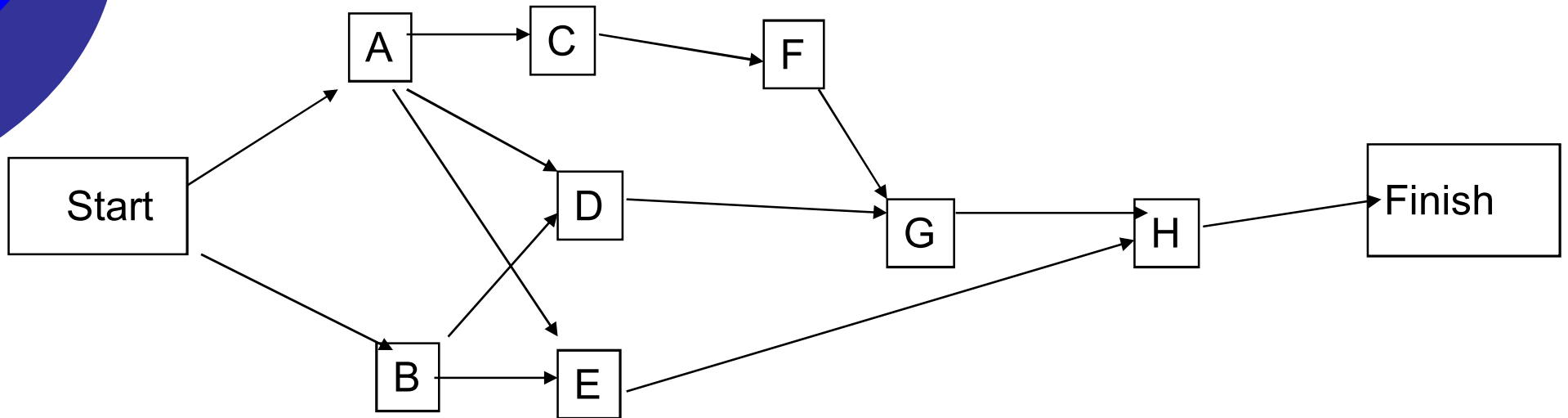
Constructing Project Networks

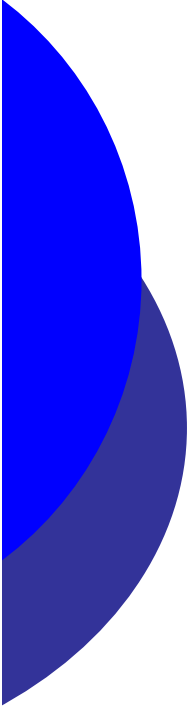
- ✚ The Mohawk Discount Store is designing a management training program for individuals at its corporate headquarters. The company wants to design a program so that trainees can complete it as quickly as possible. Important precedence relationships must be maintained between assignments or activities in the program. For example, a trainee cannot serve as an assistant to the store manager until the employee has obtained experience in the credit department and at least one sales department. The following activities are the assignments that must be completed by each program trainee. Construct the project network for this problem. (Anderson, et. Al, Chapter 10, problem 1)

Activities A – H represent actual tasks.

Activity	A	B	C	D	E	F	G	H
Immediate Predecessor	---	---	A	A, B	A,B	C	D,F	E,G

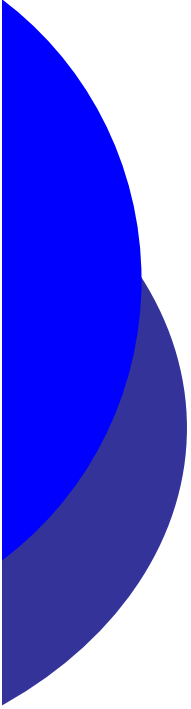
Mohawk Project Network





Project Network Discussion

- ⊕ Project networks are not unique. A project network is considered valid provided all precedence relationships are preserved.
- ⊕ Mohawks project network shows that no activities precede activities A and B. For this reason an arc goes directly from start to these activity nodes.
- ⊕ The immediate predecessors of each node is (are) displayed on the network by arcs leading from these immediate predecessors to the node.
- ⊕ Also notice that activity H is the only activity that has an arc that goes directly to the finish node. **ONLY ACTIVITIES THAT ARE NOT IMMEDIATE PREDECESSORS TO ANY OTHER NETWORK ACTIVITIES MAY HAVE A LINK DIRECTLY TO THE FINISH NODE.**



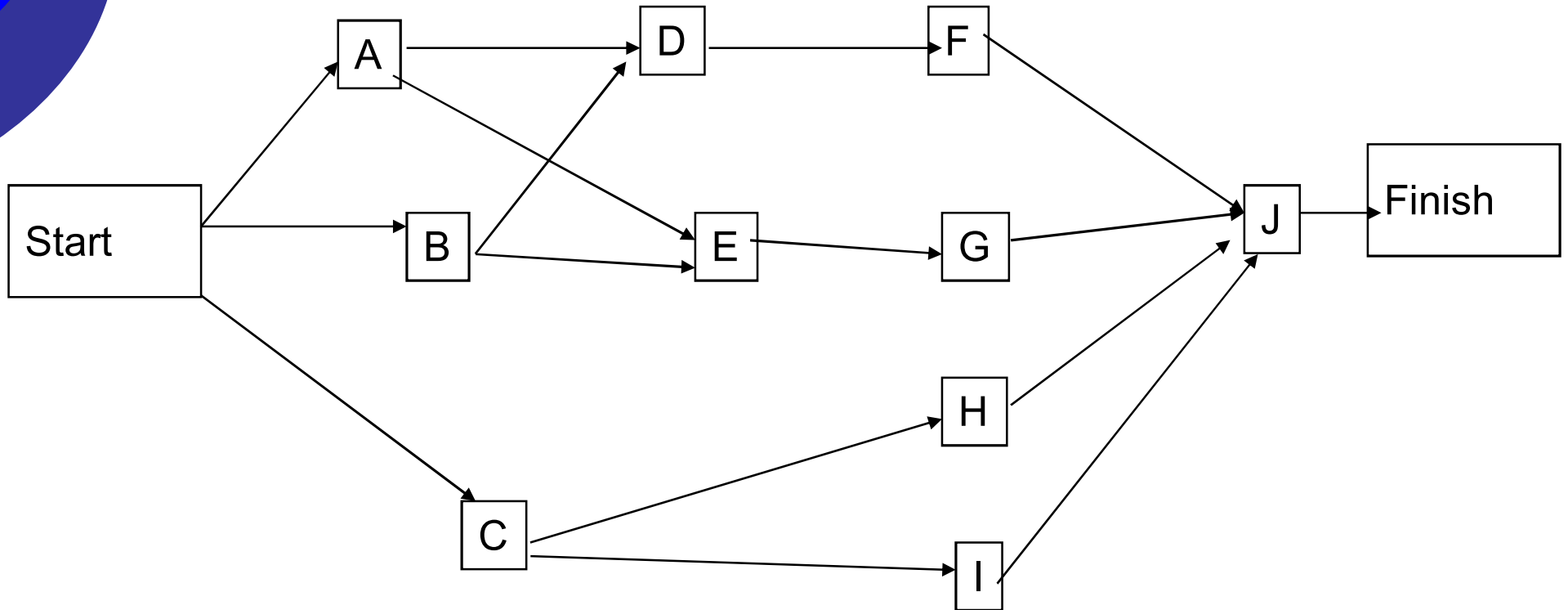
Another Project Network Example

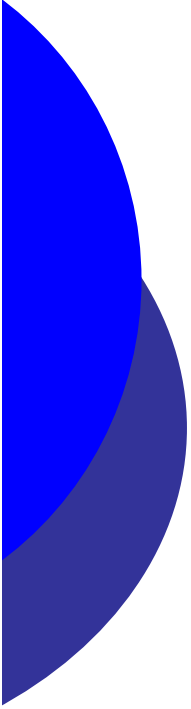
- ⊕ Bridge City Developers is coordinating the construction of an office complex. As part of the planning process, the company generated the following activity list. Draw a project network that can be used to assist in the scheduling of the project activities.

Activity	A	B	C	D	E	F	G	H	I	J
Immediate Predecessor	---	---	---	A,B	A, B	D	E	C	C	F,G,H,I

Bridge City Developers Project Network

Here is one way of depicting a Bridge City Developers Project Network

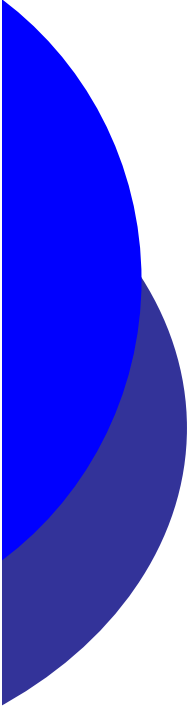




Overall Procedure for solving a Project Network

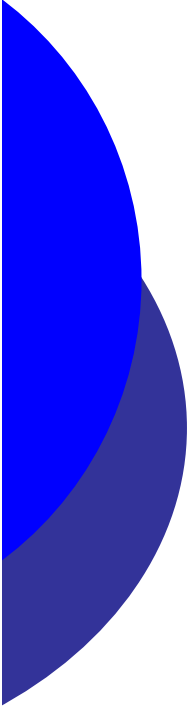
1. Determine the sequence of activities.
2. Construct the network or precedence diagram.
3. Starting from the left, compute the Early Start (ES) and Early Finish (EF) time for each activity.
4. Starting from the right, compute the Late Finish (LF) and Late Start (LS) time for each activity.
5. Find the slack for each activity.
6. Identify the Critical Path.

In the following slides will elaborate on steps 3-6.



Project Management Notation

t	Duration of an activity
ES	The earliest time an activity can start
EF	The earliest time an activity can finish ($EF = ES + t$)
LS	The latest time an activity can start and not delay the project
LF	The latest time an activity can finish and not delay the project
Slack	The extra time that could be made available to an activity without delaying the project ($Slack = LS - ES$)
Critical Path	The sequence(s) of activities with no slack

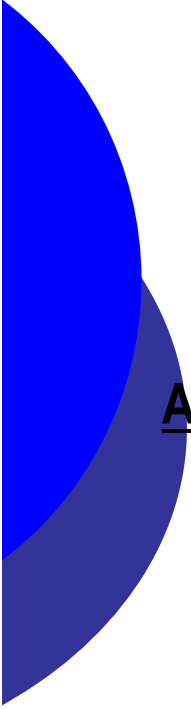


Example: Frank's Fine Floats

Frank's Fine Floats is in the business of building elaborate parade floats. Frank and his crew have a new float to build and want to use PERT/CPM to help them manage the project .

The table on the next slide shows the activities that comprise the project. Each activity's estimated completion time (in days) and immediate predecessors are listed as well.

Frank wants to know the total time to complete the project, which activities are critical, and the earliest and latest start and finish dates for each activity.

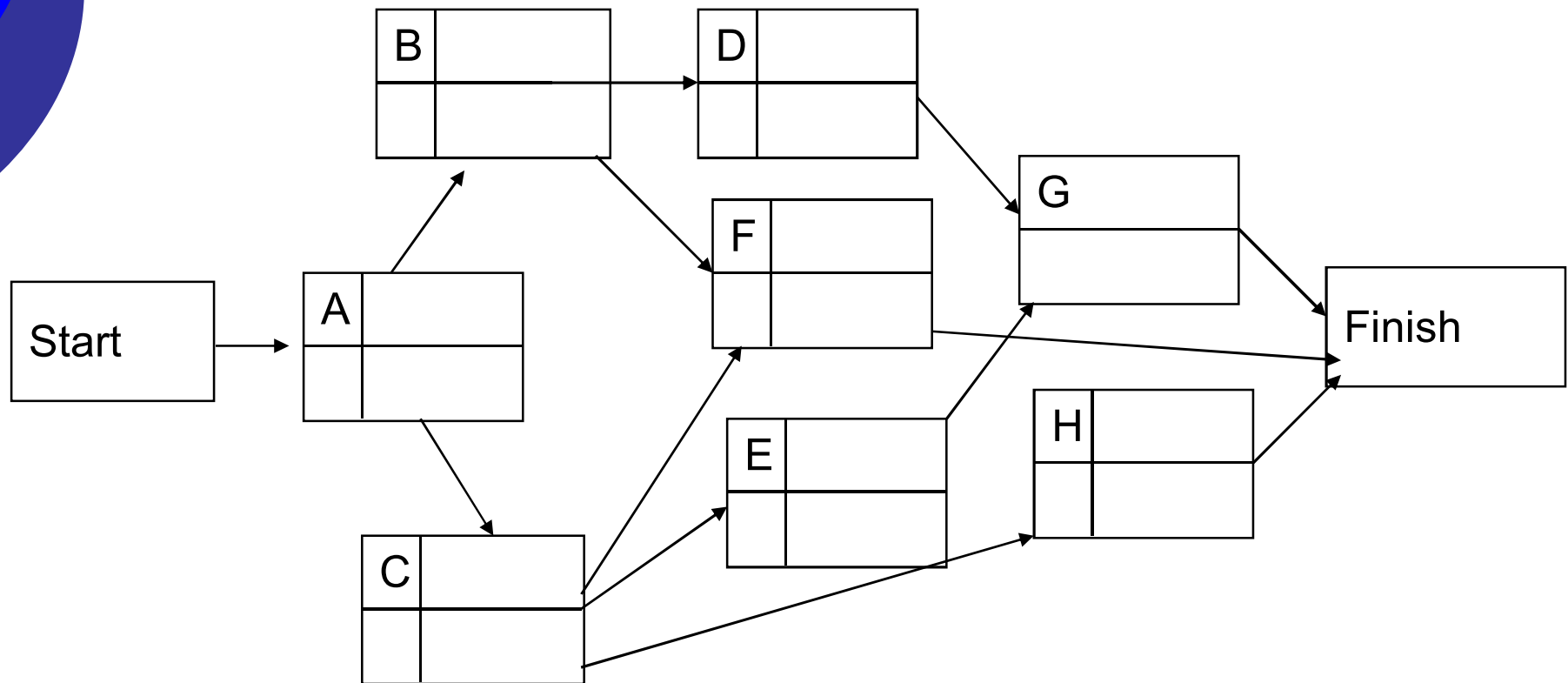


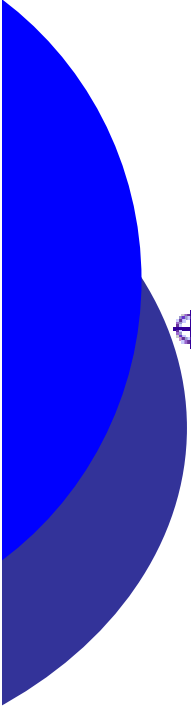
Example: Frank's Fine Floats

<u>Activity</u>	<u>Description</u>	<u>Immediate Predecessors</u>	<u>Completion Time (days)</u>
A	Initial Paperwork	---	3
B	Build Body	A	3
C	Build Frame	A	2
D	Finish Body	B	3
E	Finish Frame	C	7
F	Final Paperwork	B,C	3
G	Mount Body to Frame	D,E	6
H	Install Skirt on Frame	C	2

Example: Frank's Fine Floats

Project Network





Earliest Start and Finish Times

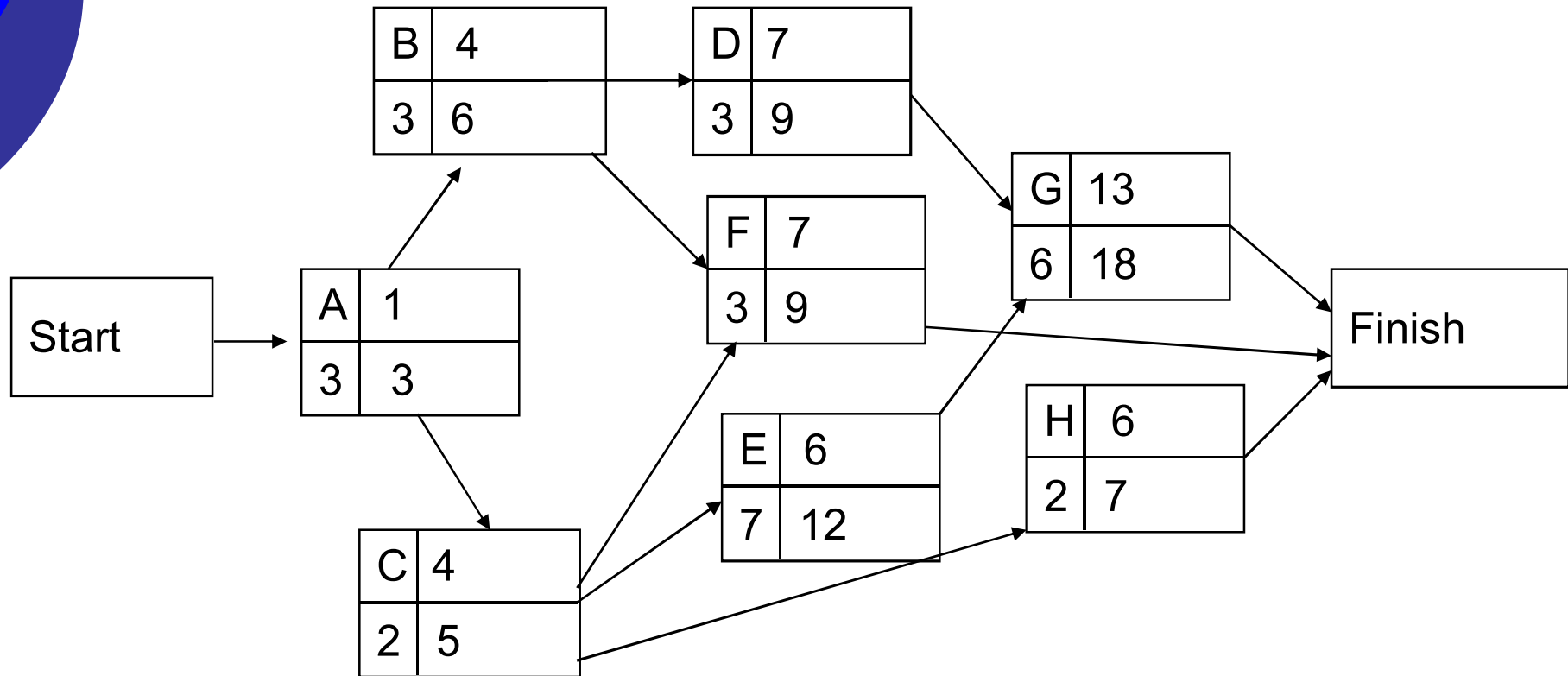
⊕ Step 3: Make a forward pass through the network as follows: For each activity i beginning at the Start node, compute:

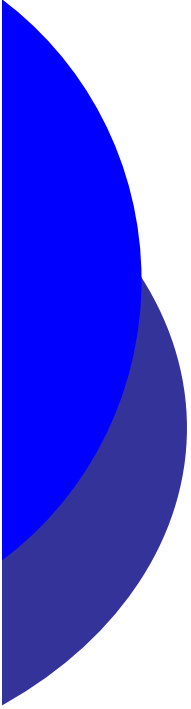
- Earliest Start Time = the maximum of the earliest finish times of all activities immediately preceding activity i . (This is 0 for an activity with no predecessors.)
- Earliest Finish Time = (Earliest Start Time) + (Time to complete activity i).

The project completion time is the maximum of the Earliest Finish Times at the Finish node.

Example: Frank's Fine Floats

⊕ Earliest Start and Finish Times





Latest Start and Finish Times

Step 4: Make a backwards pass through the network as follows:

Latest Finish Time Rule: $LF = \text{Smallest } LS \text{ of the immediate successors.}$

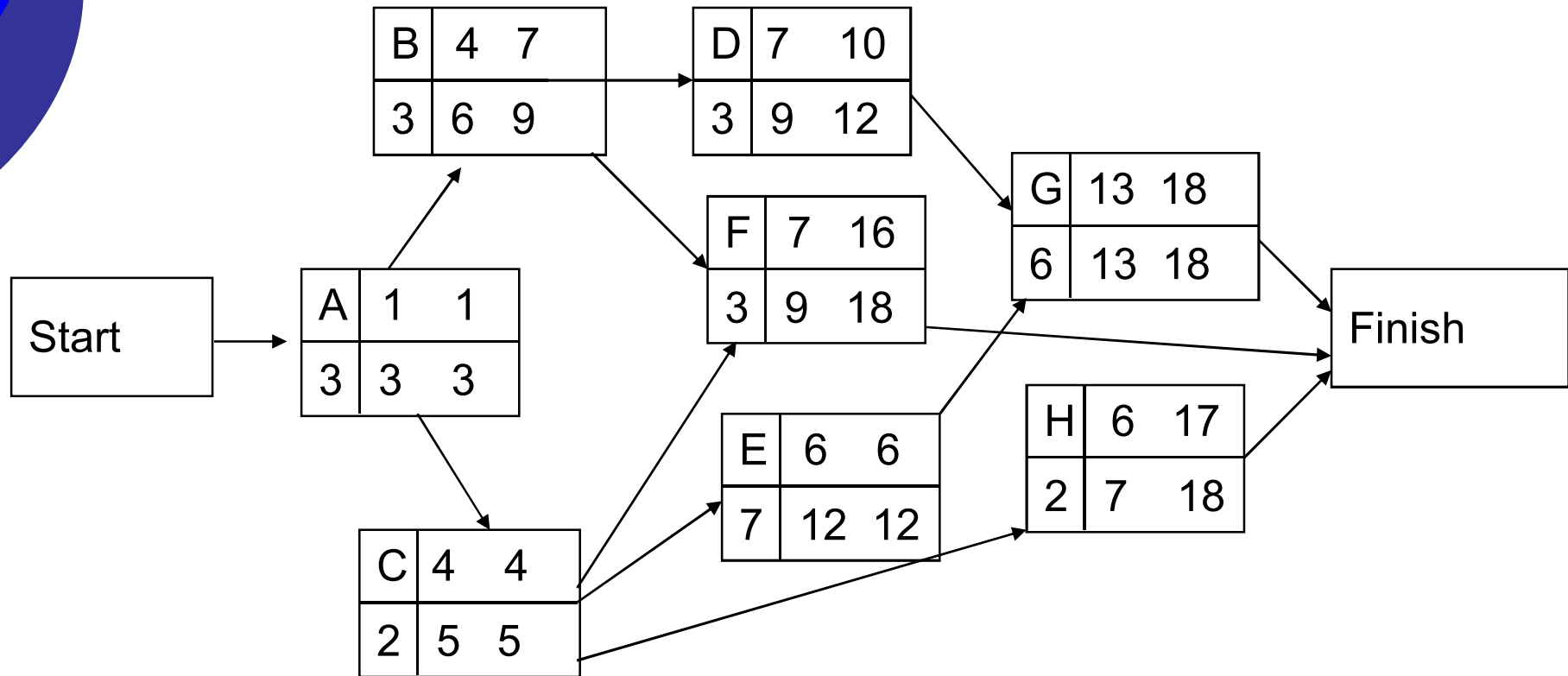
The immediate successors for a node are all nodes that immediately follow the current node.

Procedure for obtaining latest times for all activities:

- 1. For each of the activities that link directly to the finish node, set LF equal to project completion time.**
- 2. For each activity whose LF value has just been obtained, calculate $LS = LF - (\text{the time to complete the current activity})$**
- 3. For each new activity whose immediate successors now have LS values, obtain its LF by applying the latest finish time rule. Apply step 2 to calculate its LS.**
- 4. Repeat step 3 until LF and LS have been obtained for all activities.**

Example: Frank's Fine Floats

⊕ Latest Start and Finish Times

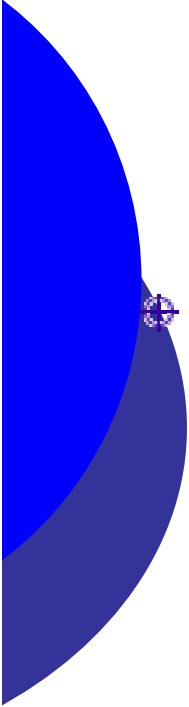




Determining the Critical Path

⊕ **Step 5: Calculate the slack time for each activity by:**

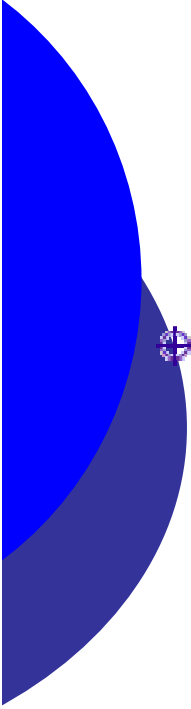
$$\begin{aligned}\text{Float} &= (\text{Latest Start}) - (\text{Earliest Start}), \text{ or} \\ &= (\text{Latest Finish}) - (\text{Earliest Finish}).\end{aligned}$$



Example: Frank's Fine Floats

Activity Slack Time

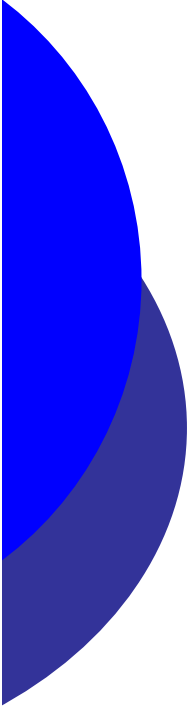
<u>Activity</u>	<u>ES</u>	<u>EF</u>	<u>LS</u>	<u>LF</u>	<u>Float</u>
A	0	3	0	3	0 (critical)
B	3	6	6	9	3
C	3	5	3	5	0 (critical)
D	6	9	9	12	3
E	5	12	5	12	0 (critical)
F	6	9	15	18	9
G	12	18	12	18	0 (critical)
H	5	7	16	18	11



Example: Frank's Fine Floats

Determining the Critical Path

- A critical path is a path of activities, from the Start node to the Finish node, with 0 float times.
- Critical Path: A – C – E – G
- The project completion time equals the sum of the duration times of all activities along the critical path.
- Project Completion Time: 18 days



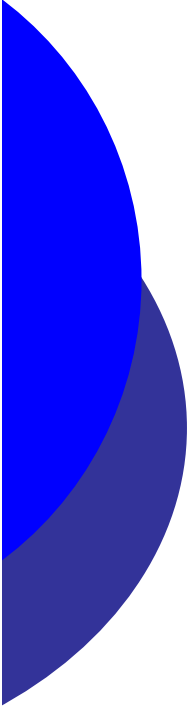
Frank's Fine Floats: Network Results

✚ The table on slide 23 reveals that the following schedule should be followed in order for the project to completed in 18 days.

- Activity A: Must begin on day 1 and be finished by day 3.
- Activity B: May begin between day 4-6, and must be completed by end of day 9.
- Activity C: Must begin on day 4 and be finished by end of day 5.

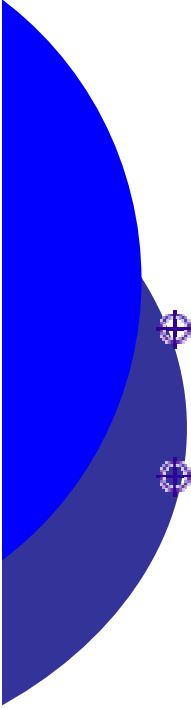
And so on.....

The project must finish by end of day 18.



PERT/CPM WITH UNCERTAIN ACTIVITY TIMES

- ⊕ Experience and historical data can be used for projects that have been completed in the past (such as home and apartment construction) to provide accurate activity time estimates.
- ⊕ In many cases, however, projects are new or unique and activity times are uncertain. In these cases estimating activity times could be difficult.
- ⊕ When there is uncertainty associated with activity times, they are often best described by a range of possible values instead of one specific time estimate.
- ⊕ Uncertain activity times are treated as random variables with associated probability distributions. These distributions allow us to form probability statements about the likelihood of meeting a specific completion date.
- ⊕ Three time estimates are collected for each activity to incorporate the uncertainty.

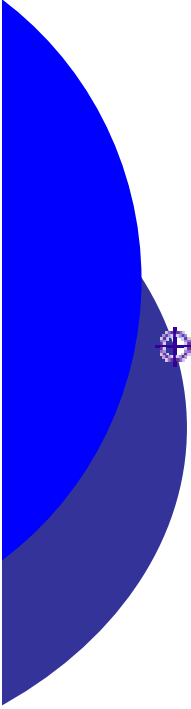


Uncertain Activity Times

- ⊕ In the three-time estimate approach, the time to complete an activity is assumed to follow a Beta distribution.
- ⊕ An activity's mean completion time is:

$$t = \frac{(a + 4m + b)}{6}$$

- a = the optimistic completion time estimate
- b = the pessimistic completion time estimate
- m = the most likely completion time estimate



Uncertain Activity Times

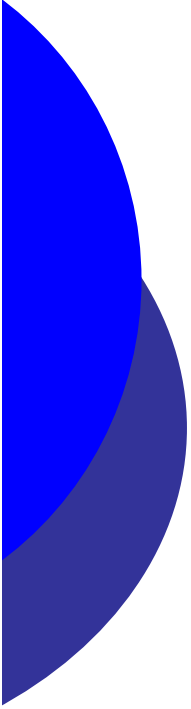
⊕ An activity's completion time variance is:

$$\left(\frac{(b - a)}{6} \right)^2$$

a = the optimistic completion time estimate

b = the pessimistic completion time estimate

m = the most likely completion time estimate



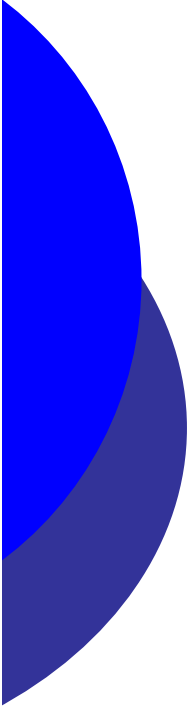
Uncertain Activity Times

- ⊕ In the three-time estimate approach, the critical path is determined as if the mean times for the activities were fixed times.
- ⊕ The overall project completion time is assumed to have a normal distribution with mean equal to the sum of the means along the critical path and variance equal to the sum of the variances along the critical path.

Example: ABC Associates

Consider the following project:

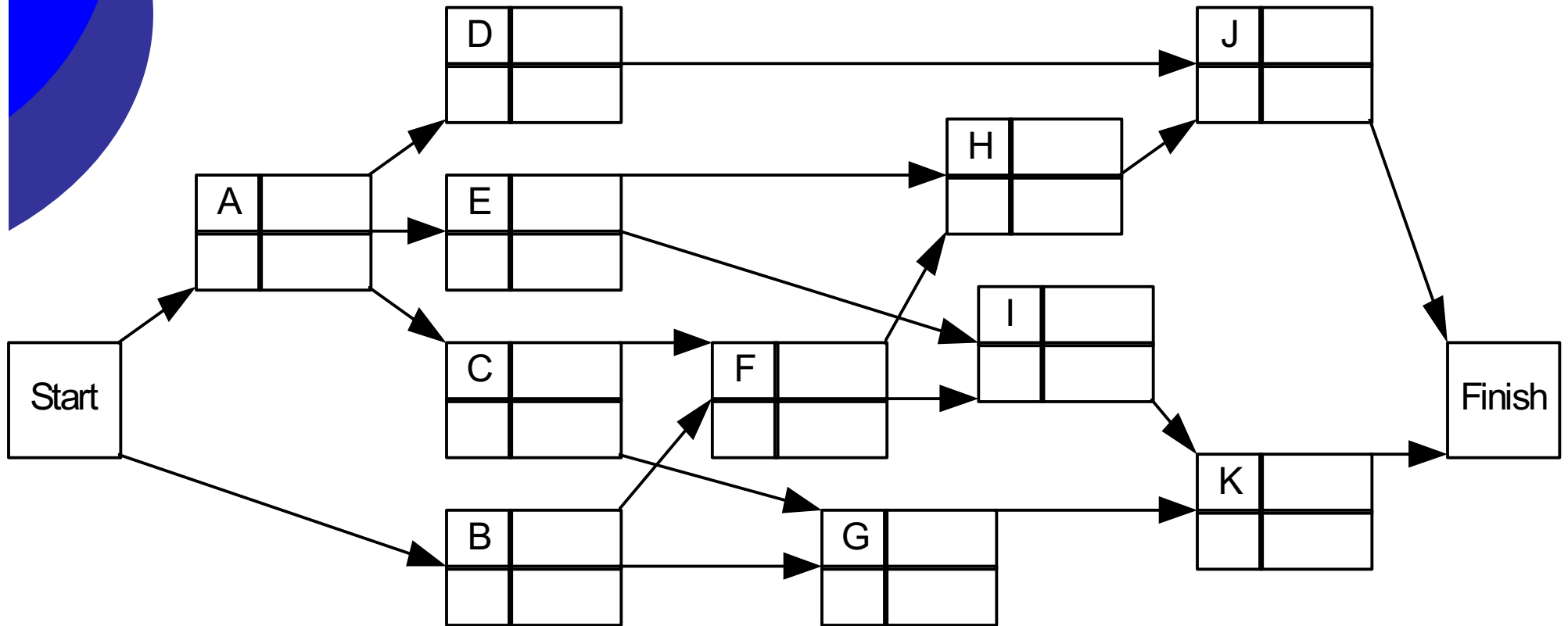
<u>Activity</u>	<u>Immed. Predec.</u>	<u>Optimistic Time (Hr.)</u>	<u>Most Likely Time (Hr.)</u>	<u>Pessimistic Time (Hr.)</u>
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B,C	3	4	5
G	B,C	1	1.5	5
H	E,F	5	6	7
I	E,F	2	5	8
J	D,H	2.5	2.75	4.5
K	G,I	3	5	7



Example: ABC Associates

- ⊕ What is the earliest completion date?
- ⊕ What is the critical path?
- ⊕ If management has set a completion deadline for 24 hours, what is the probability that they will meet this deadline?

ABC Associates Project Network



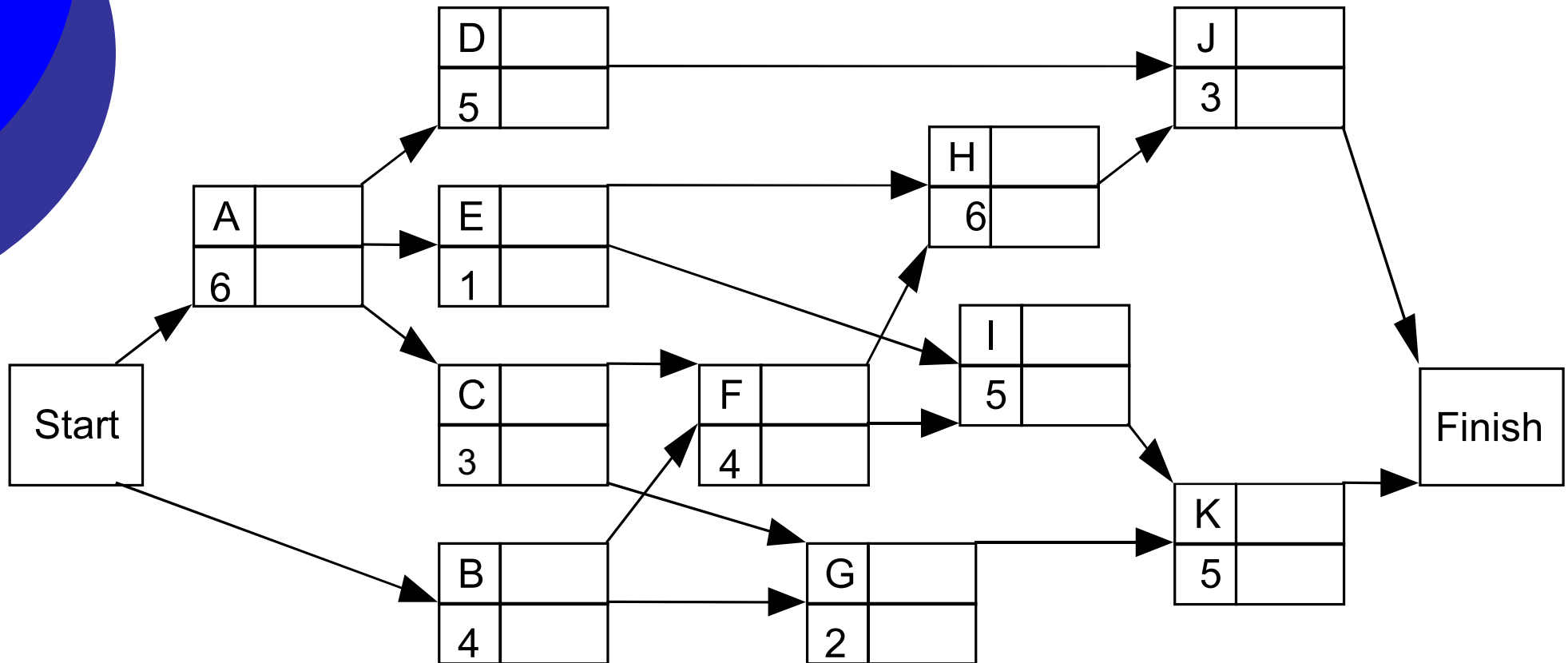
Example: ABC Associates

Activity Expected Times and Variances

$$t = (a + 4m + b)/6 \quad \sigma^2 = ((b-a)/6)^2$$

<u>Activity</u>	<u>Expected Time</u>	<u>Variance</u>
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

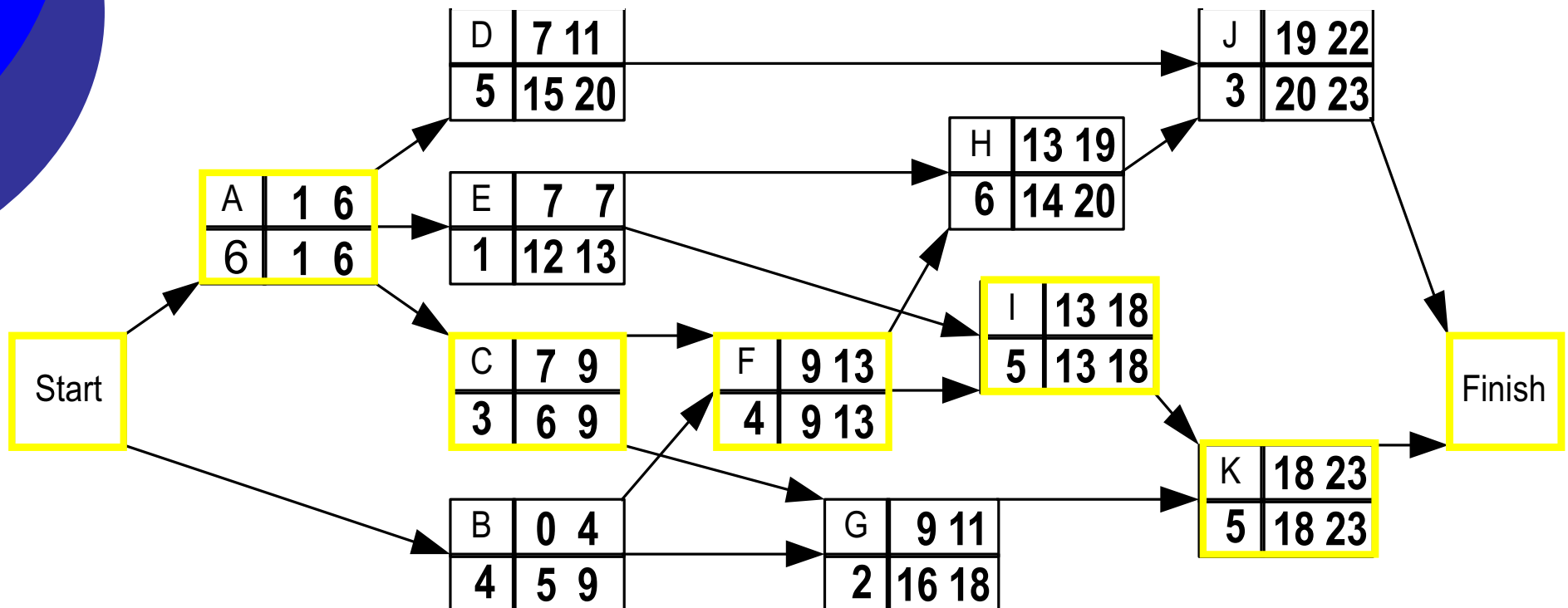
Example: ABC Associates

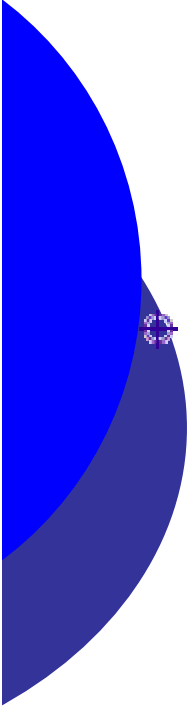


Complete a forward and backward pass to fill in the above network.

Example: ABC Associates

⊕ Critical Path (A-C-F-I-K)

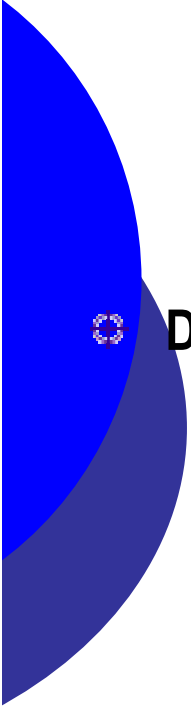




Example: ABC Associates

Earliest/Latest Times and Slack

<u>Activity</u>	<u>ES</u>	<u>EF</u>	<u>LS</u>	<u>LF</u>	<u>Float</u>
A	0	6	0	6	0 *
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0 *
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	23	18	23	0 *



Example: ABC Associates

❏ Determining the Critical Path

- The critical path is a path of activities, from the Start node to the Finish node, with 0 slack times.
- Critical Path: A – C – F – I – K
- The project completion time equals the sum of the duration times of all activities along the critical path.
- Project Completion Time: 23 hours

Example: ABC Associates

⊕ Probability the project will be completed within 24 hrs: $P(X < 24)$

The mean completion time $E(T)$ = the sum of the duration times of all activities along the critical path. Thus $E(T) = 23$.

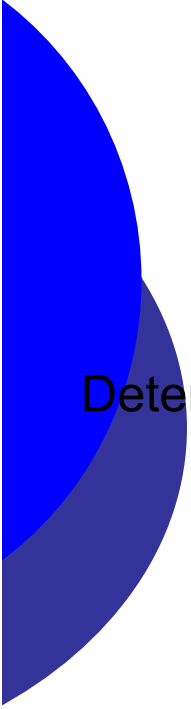
$$\begin{aligned}\sigma^2 &= \sigma_A^2 + \sigma_C^2 + \sigma_F^2 + \sigma_H^2 + \sigma_K^2 \\ &= 4/9 + 0 + 1/9 + 1 + 4/9 \\ &= 2\end{aligned}$$

$$\sigma = 1.414$$

T = Completion time
 $E(T)$ = Expected completion Time

PERT analysis

- Draw the network.
- Analyze the paths through the network and find the critical path.
- The length of the critical path is the mean of the project duration probability distribution which is assumed to be normal
- ⊕ The standard deviation of the project duration probability distribution is computed by adding the variances of the critical activities (all of the activities that make up the critical path) and taking the square root of that sum
- ⊕ Probability computations can now be made using the normal distribution table.



Probability computation

Determine probability that project is completed within specified time

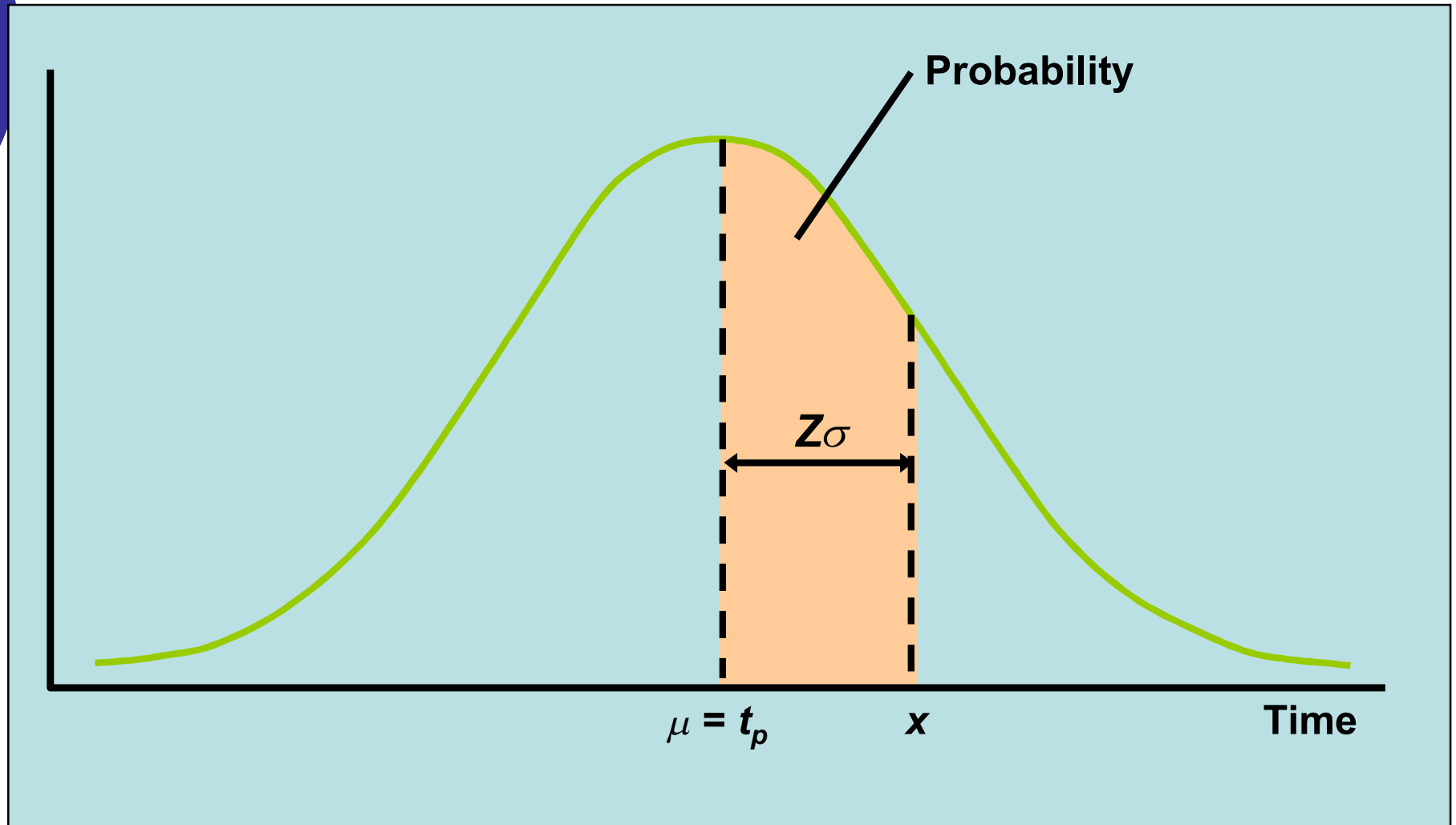
$$Z = \frac{X - \mu}{\sigma}$$

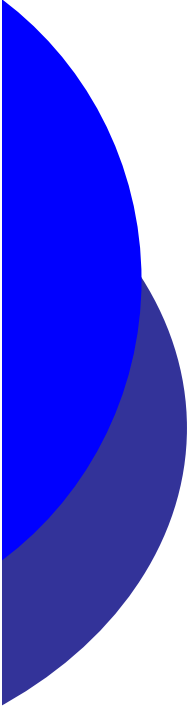
where $\mu = t_p$ = project mean time

σ = project standard mean time

x = (proposed) specified time

Normal Distribution of Project Time





PERT Example

$$\begin{aligned}V_{\text{path}} &= V_A + V_C + V_F + V_I + V_K \\&= 4/9 + 0 + 1/9 + 1 + 4/9 \\&= 2\end{aligned}$$

$$\sigma_{\text{path}} = 1.414$$

$$z = (24 - 23)/\sigma = (24-23)/1.414 = .71$$

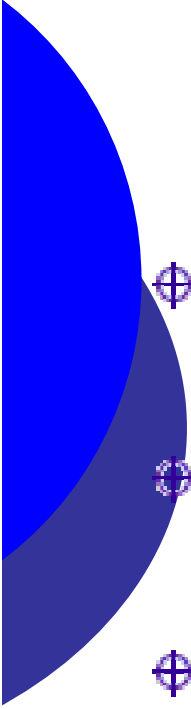
From the Standard Normal Distribution table:

$$P(z \leq .71) = .5 + .2612 = .7612$$



Cost consideration in project

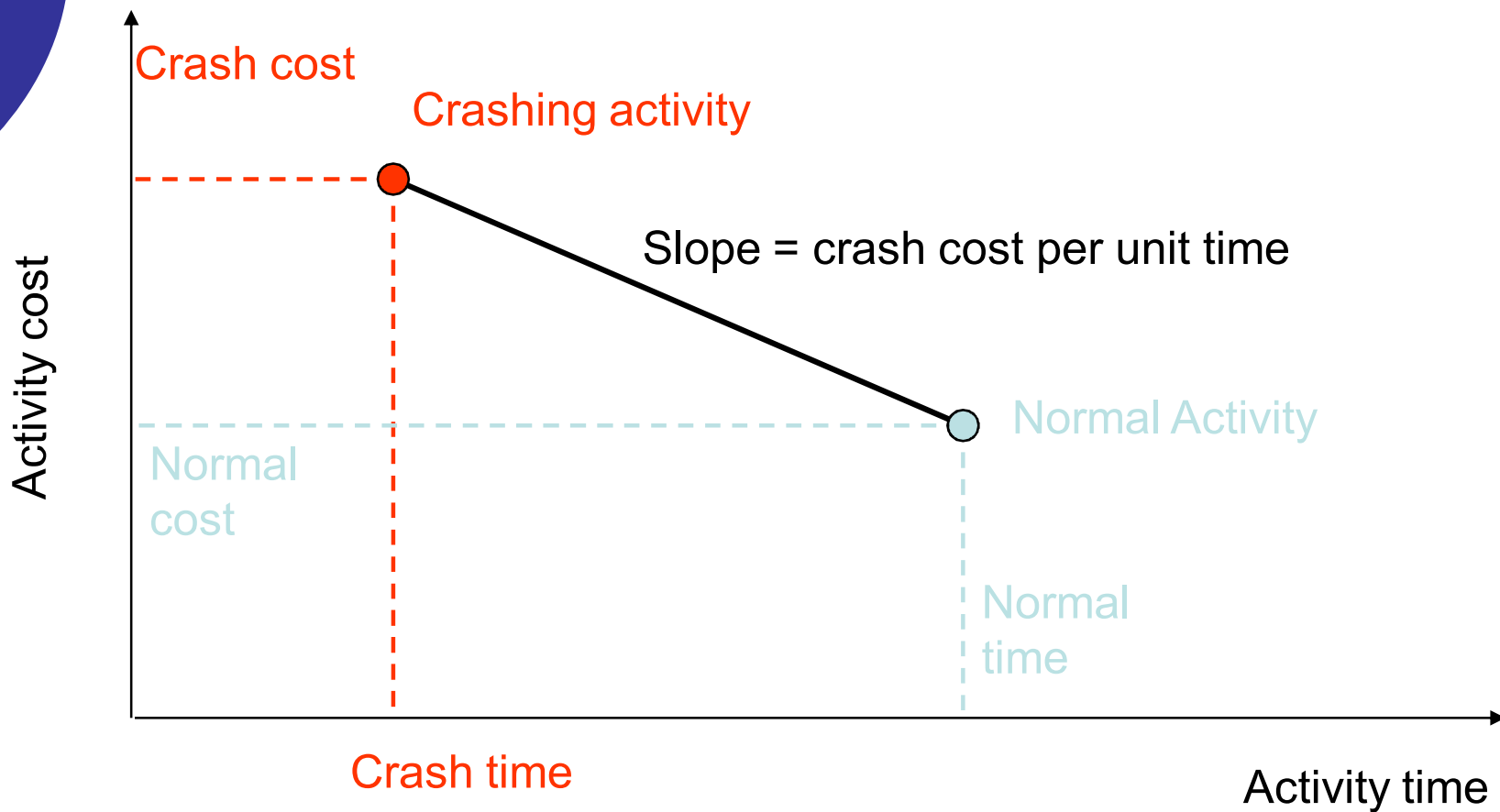
- Project managers may have the option or requirement to crash the project, or accelerate the completion of the project.
- This is accomplished by reducing the length of the critical path(s).
- The length of the critical path is reduced by reducing the duration of the activities on the critical path.
- ⊕ If each activity requires the expenditure of an amount of money to reduce its duration by one unit of time, then the project manager selects the least cost critical activity, reduces it by one time unit, and traces that change through the remainder of the network.
- ⊕ As a result of a reduction in an activity's time, a new critical path may be created.
- ⊕ When there is more than one critical path, each of the critical paths must be reduced.
- ⊕ If the length of the project needs to be reduced further, the process is repeated.



Project Crashing

- ⊕ **Crashing**
 - reducing project time by expending additional resources
- ⊕ **Crash time**
 - an amount of time an activity is reduced
- ⊕ **Crash cost**
 - cost of reducing activity time
- ⊕ **Goal**
 - reduce project duration at minimum cost

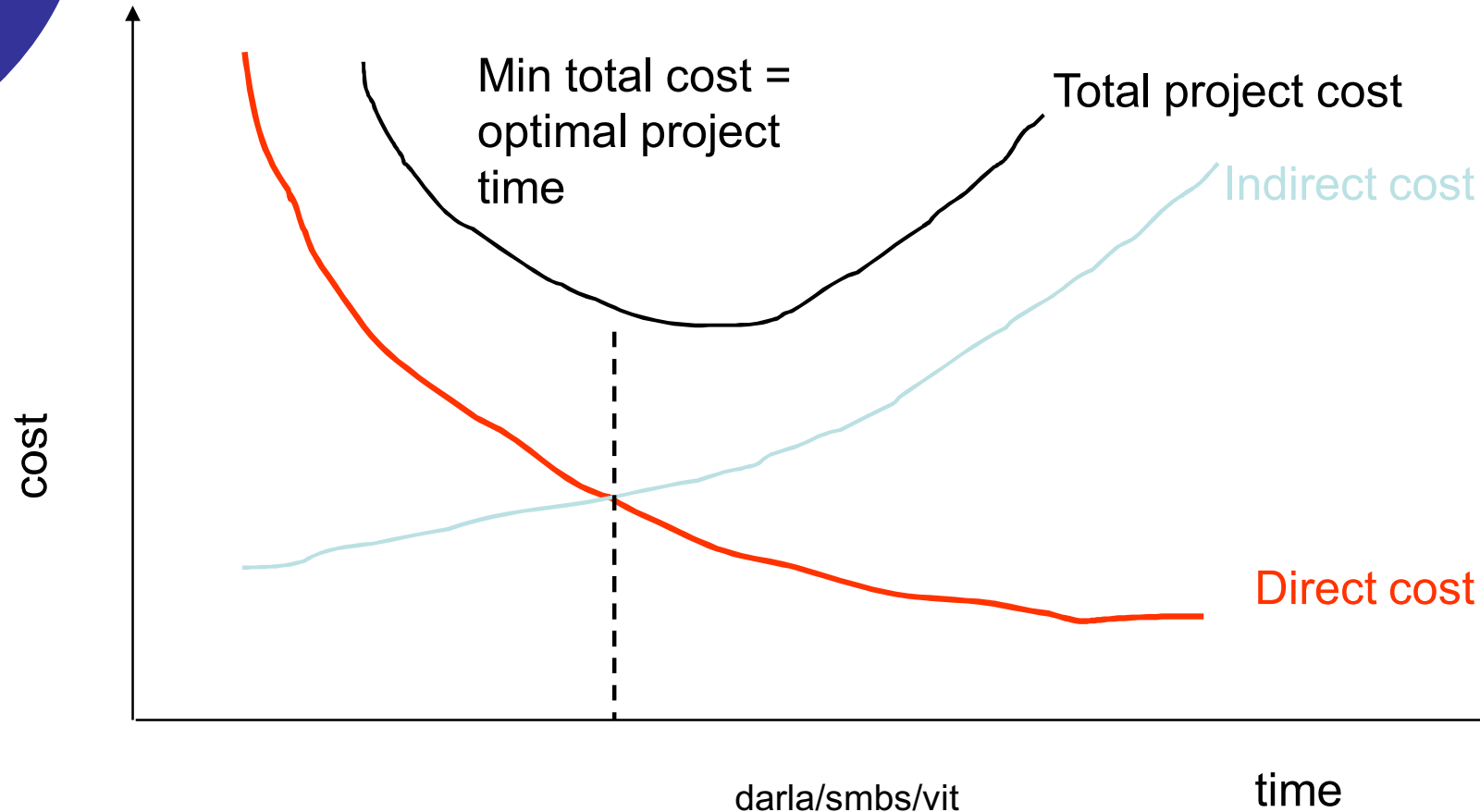
Activity crashing



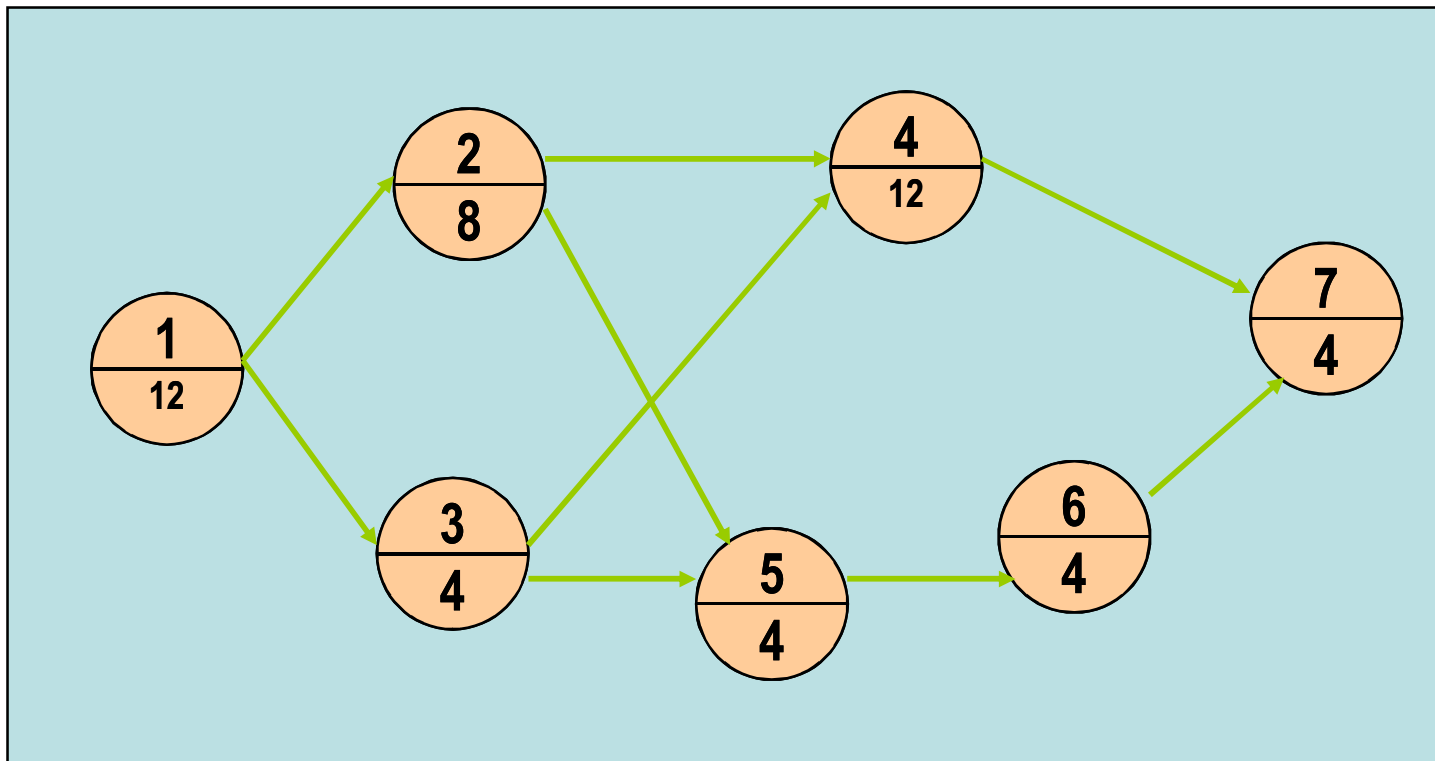
Time-Cost Relationship

- Crashing costs increase as project duration decreases
- Indirect costs increase as project duration increases
- Reduce project length as long as crashing costs are less than indirect costs

Time-Cost Tradeoff

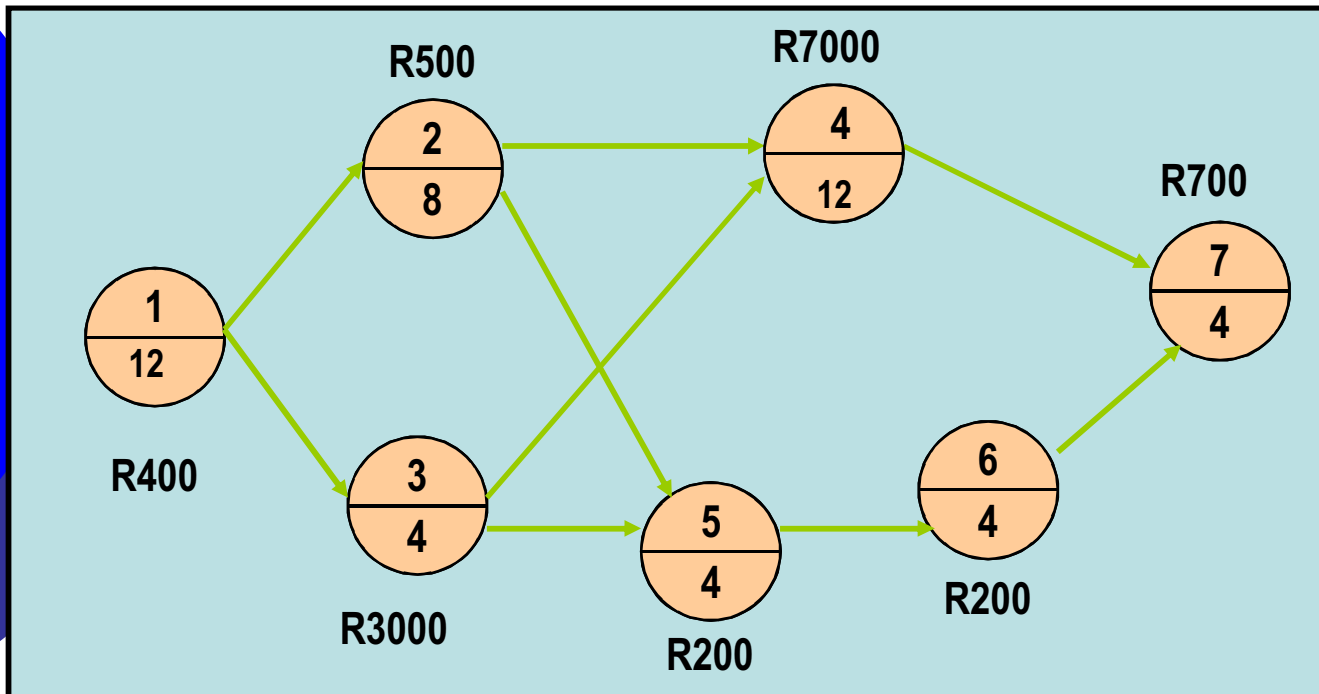


Project Crashing example



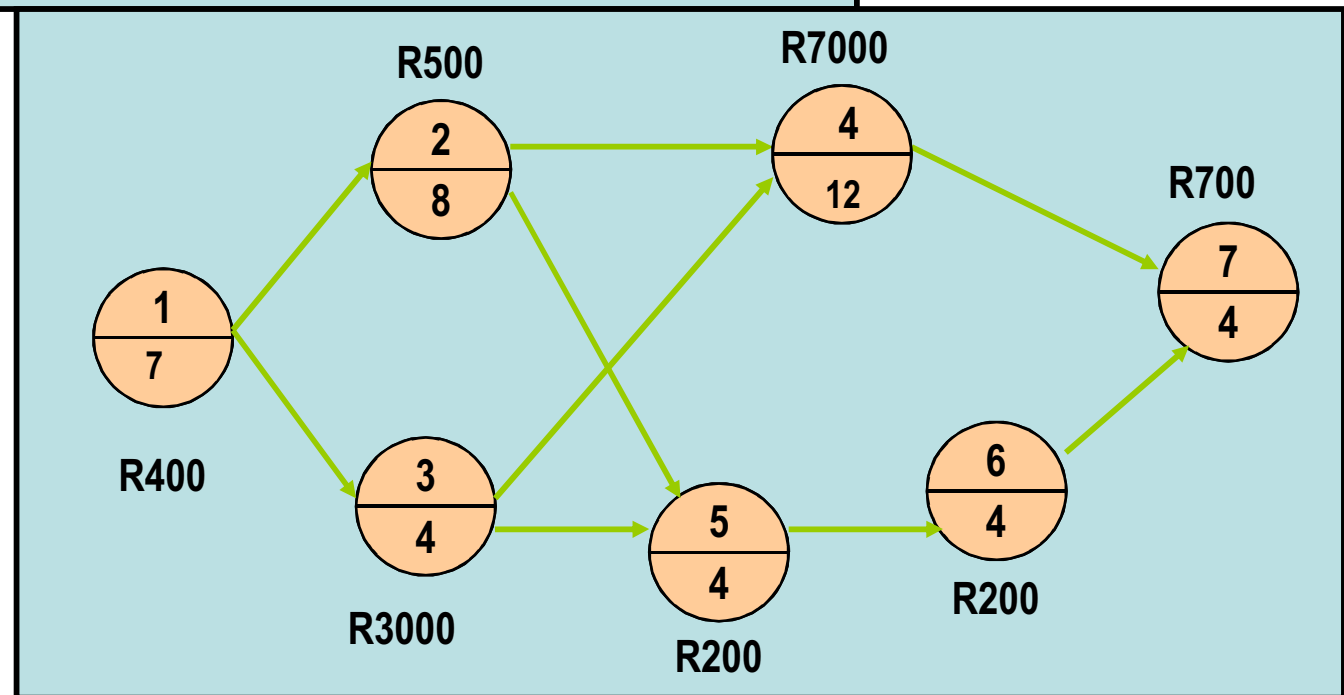
Time Cost data

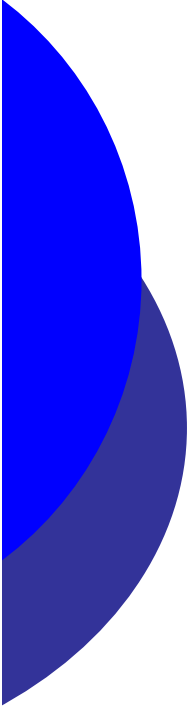
Activity	Normal time	Normal cost Rs	Crash time	Crash cost Rs	Allowable crash time	slope
1	12	3000	7	5000	5	400
2	8	2000	5	3500	3	500
3	4	4000	3	7000	1	3000
4	12	50000	9	71000	3	7000
5	4	500	1	1100	3	200
6	4	500	1	1100	3	200
7	4	1500	3	22000	1	7000
		75000		110700		



To.....

Project
duration = 31
Additional cost =
R2000





Benefits of CPM/PERT

- ⊕ Useful at many stages of project management
- ⊕ Mathematically simple
- ⊕ Give critical path and slack time
- ⊕ Provide project documentation
- ⊕ Useful in monitoring costs



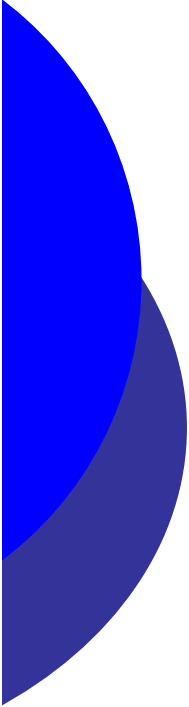
CPM/PERT can answer the following important questions:

- How long will the entire project take to be completed? What are the risks involved?
- Which are the critical activities or tasks in the project which could delay the entire project if they were not completed on time?
- Is the project on schedule, behind schedule or ahead of schedule?
- If the project has to be finished earlier than planned, what is the best way to do this at the least cost?

Limitations to CPM/PERT

- ⊕ Clearly defined, independent and stable activities
- ⊕ Specified precedence relationships
- ⊖ Over emphasis on critical paths
- ⊖ Deterministic CPM model
- ⊕ Activity time estimates are subjective and depend on judgment
- ⊕ PERT assumes a beta distribution for these time estimates, but the actual distribution may be different
- ⊕ PERT consistently underestimates the expected project completion time due to alternate paths becoming critical

To overcome the limitation, Monte Carlo simulations can be performed on the network to eliminate the optimistic bias



Computer Software for Project Management

- ⊕ **Microsoft Project (Microsoft Corp.)**
- ⊕ **MacProject (Claris Corp.)**
- ⊕ **PowerProject (ASTA Development Inc.)**
- ⊕ **Primavera Project Planner (Primavera)**
- ⊕ **Project Scheduler (Scitor Corp.)**
- ⊕ **Project Workbench (ABT Corp.)**

Practice Example

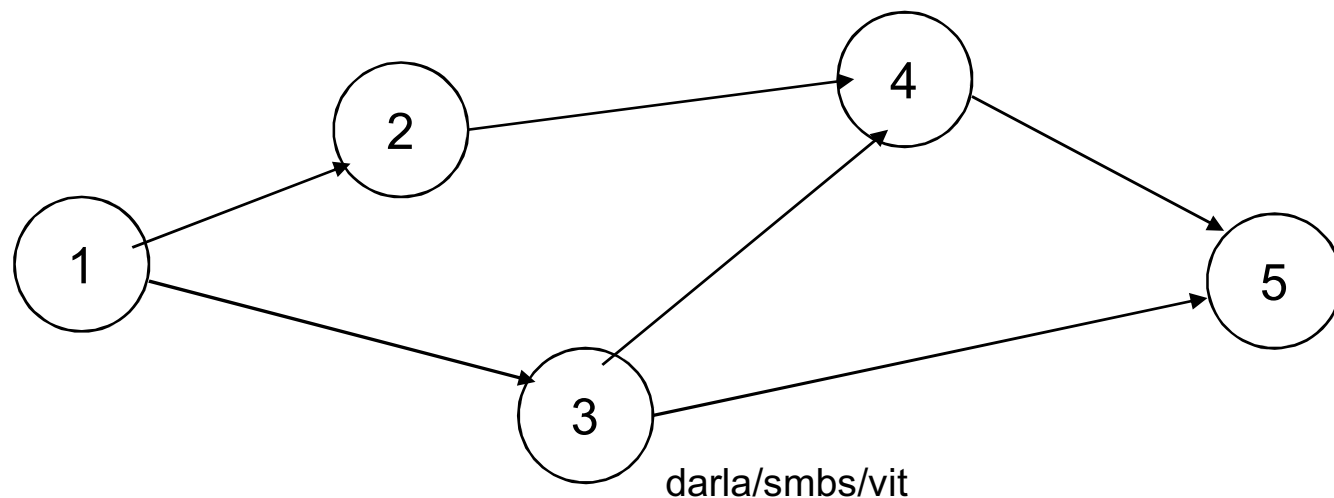
A social project manager is faced with a project with the following activities:

Activity Description	Duration
Social work team to live in village	5w
Social research team to do survey	12w
Analyse results of survey	5w
Establish mother & child health program	14w
Establish rural credit programme	15w
Carry out immunization of under fives	4w

Draw network diagram and show the critical path. Calculate project duration.

Practice problem

Activity	Description	Duration
1-2	Social work team to live in village	5w
1-3	Social research team to do survey	12w
3-4	Analyse results of survey	5w
2-4	Establish mother & child health program	14w
3-5	Establish rural credit programme	15w
4-5	Carry out immunization of under fives	4w



Example: EarthMover, Inc.

EarthMover is a manufacturer of road construction equipment including pavers, rollers, and graders. The company is faced with a new project, introducing a new line of loaders. Management is concerned that the project might take longer than 26 weeks to complete without crashing some activities.



Example: EarthMover, Inc.

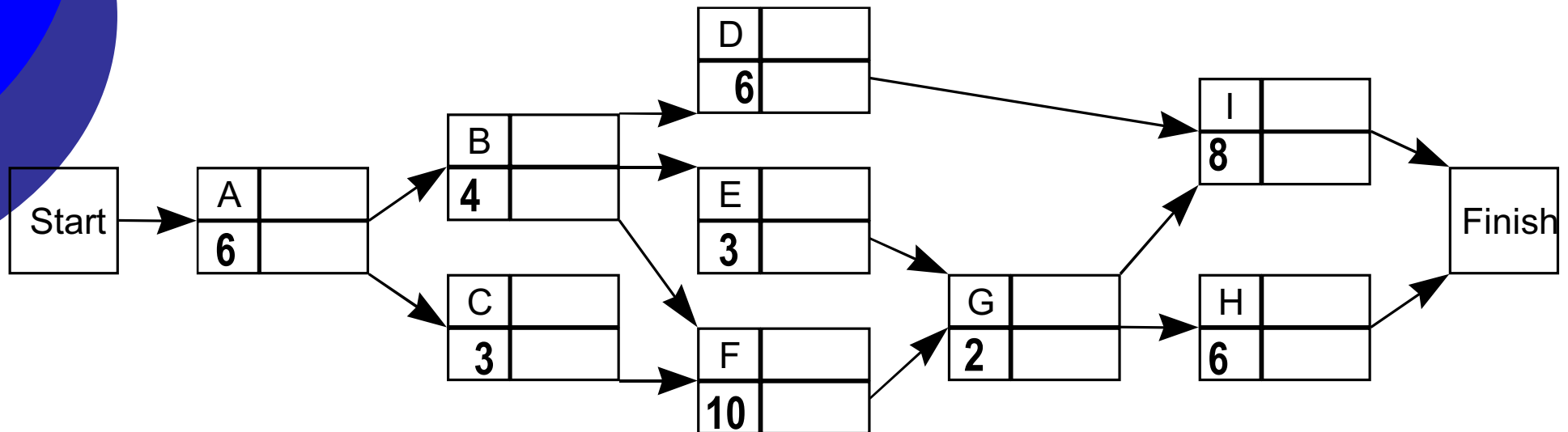


<u>Activity</u>	<u>Description</u>	<u>Immediate Predecessors</u>	<u>Completion Time (wks)</u>
A	Study Feasibility	---	6
B	Purchase Building	A	4
C	Hire Project Leader	A	3
D	Select Advertising Staff	B	6
E	Purchase Materials	B	3
F	Hire Manufacturing Staff	B,C	10
G	Manufacture Prototype	E,F	2
H	Produce First 50 Units	G	6
I	Advertise Product	D,G	8

Example: EarthMover, Inc.



PERT Network



Example: EarthMover, Inc.



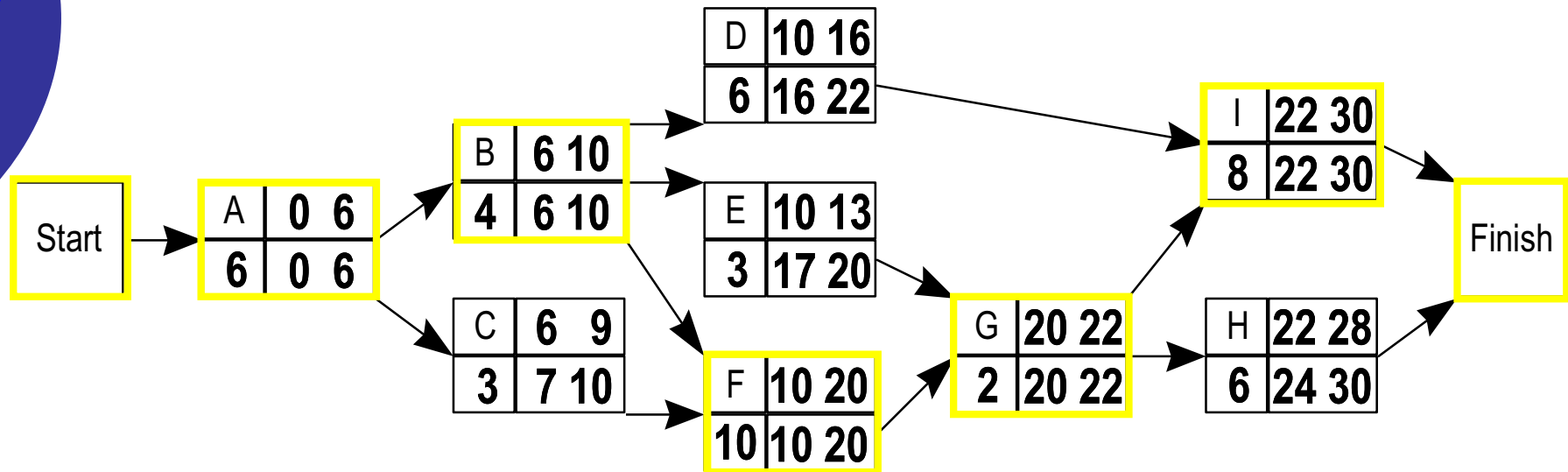
Earliest/Latest Times

<u>Activity</u>	<u>ES</u>	<u>EF</u>	<u>LS</u>	<u>LF</u>	<u>Float</u>
A	0	6	0	6	0 *
B	6	10	6	10	0 *
C	6	9	7	10	1
D	10	16	16	22	6
E	10	13	17	20	7
F	10	20	10	20	0 *
G	20	22	20	22	0 *
H	22	28	24	30	2
I	22	30	22	30	0 *

Example: EarthMover, Inc.



⊕ Critical Activities



Example: EarthMover, Inc.

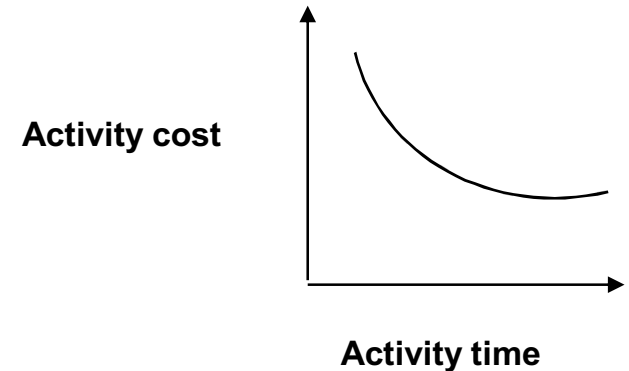


⊕ Crashing

The completion time for this project using normal times is 30 weeks. Which activities should be crashed, and by how many weeks, in order for the project to be completed in 26 weeks?

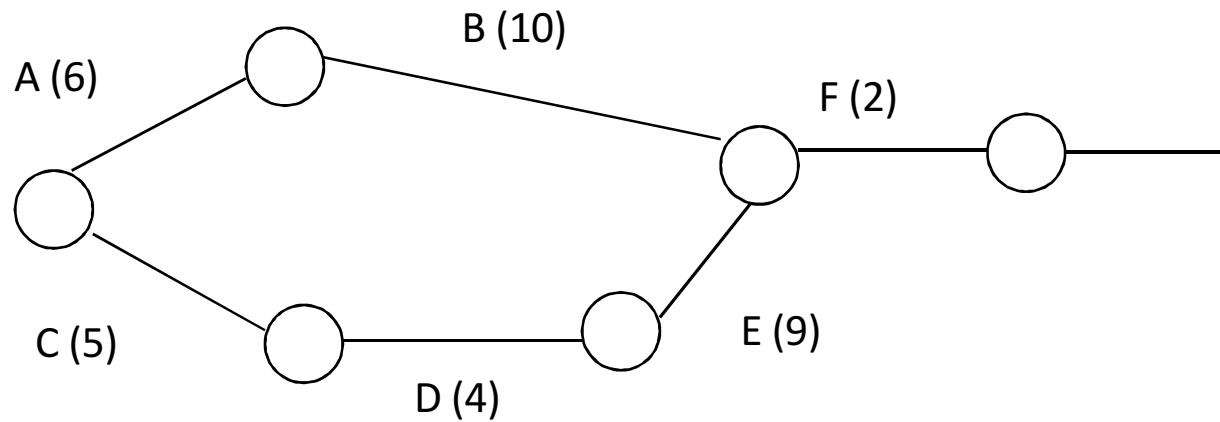
CRASHING OF PROJECT

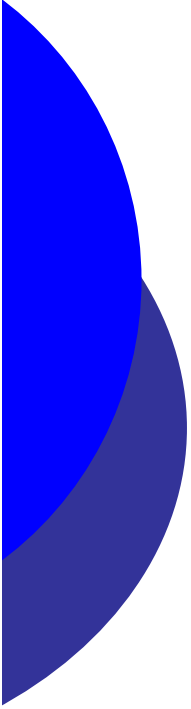
- ✚ The time cost relationship can be visualized graphically in the form of a time versus cost curve which is for a limited portion at least, sloping downwards. If the activity takes more time than the cost increases due to inefficient use of resources. Also, if the activity has to be completed in a less time, then more funds have to be spent, which implies more costs.



⊕ Example-

Activity	Normal time	Crash time	Crash cost/day
a	6	6	---
b	10	8	500
c	5	4	300
d	4	1	700
e	9	7	600
f	2	1	800





a-b-f = 18 days

Critical path □ c-d-e-f = 20 days

Arrange activities in increasing order of cost

c 300

e 600

d 700

f 800

Crash c by one day, critical path = 19 days

Cost= 300

Save= 700

Crash e by one day, critical path = 18 days

Cost= 900 (300 +600)

Save= 1100

Now both paths have become critical with duration of 18 days, to further crash the project we have to crash 1 day each from both the paths in that case either we can choose 1 day of either activities from both the path or choosing activity common in both path.

Crash f by one day, critical path = 17 days

Cost= 1700

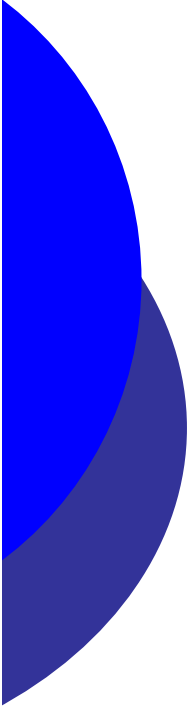
Save= 3000

Crash b and e by one day each, critical path = 16 days

Cost= 2800

Save= 2900

Project cannot be crashed further.



<u>PERT</u>	<u>CPM</u>
A probabilistic model with uncertainty in activity duration. The duration of each activity is normally computed from multiple time estimates with a view to take into account time uncertainty. These estimates are ultimately used to arrive at the probability of achieving any given scheduled date of project completion.	A deterministic model with well known activity (single) times based upon the past experience, it therefore, does not deal with uncertainty in time.
It is said to be event oriented as the results of analysis are expressed in terms of events or distinct points in time indicative of progress.	It is activity oriented as the results of calculations are considered in terms of activities or operations of the project.
The use of dummy activities is required for representing the proper sequencing.	The use of dummy activities is not necessary.
It is used for repetitive jobs	It is used for non-repetitive jobs.
It is applied mainly for planning and scheduling research programmes.	It is used for construction and business problems.
PERT analysis does not usually consider costs.	CPM deals with costs of project schedules and their minimization. The concept of crashing is applied mainly to CPM models.